Reading E: Net Present Value-Consistent Investment Criteria Based on Accruals: A Generalisation of the Residual Income-Identity

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Abstract

Abstract: In recent years, many firms have favoured residual income for value based management. One main argument for this measure is its identity with the net present value rule and that this compatibility with the net present value rule holds true for all possible depreciation schedules selected. In this article, we analyse whether there are other, undiscussed, accrual accounting numbers that enable net present value consistent investment decisions for all possible depreciation schedules. Our analysis provides an if-and only-if characterisation of the entire class of net present value-consistent investment criteria, based on accounting information. This provides new insights into the residual income concept, hurdle rates, opening and closing error conditions achieved by applying more common performance measure structures, and allocation rules. Moreover, our analysis shows the limits of constructing such investment criteria.

1. Introduction

(i) Definition of the Analysed Problem

For decades, much discussion in the area of management control has focused on the extent to which accrual accounting numbers support optimal investment decision making. Typically, this branch of literature considers accounting measures that are designed such that an investment centre manager who makes his investment decision by means of discounting accrual accounting numbers will make the optimal investment decision for central management. Thereby, the measures are judged against the criterion of whether they are compatible with the net present value rule (npv-rule) (see, for example, Solomons, 1965, p. 277; Egginton, 1995, p. 201; Bromwich and Walker, 1998, p. 394; Grinyer, 2000, p. 4; and Stark, 2000, p. 314).

Business research has long recognised that residual income measures are compatible with the net present value rule. Thereby, residual income is defined as the profit minus an interest charge on the book value of the investment. Preinreich (1938), Hicks (1946), Edwards and Bell (1961), Kay (1976) and Peasnell (1981 and 1982) have shown that the following residual income identity (ri-identity):

\[
\sum_{t=1}^{T} p_t c_t(t) - I = \sum_{t=1}^{T} p_t (c_t(t) - d_t I - r b_{t-1} I)
\]

is true, where \(p_t := (1 + r)^{-t}\) are exogenously given discount factors; \(r\) the costs of capital in period \(t\), \(I\) the investment at time \(t\), \(c_t(\cdot)\) the cash flows of the projects at time \(t\), \(d_t\) the depreciation at period \(t\) and \(b_{t-1} I\) the book value of the project of the period \(t-1\) \((t = 1, \ldots, T)\). To derive this identity, the following assumptions are made (see, for example, Peasnell, 1981, p. 53 and 1982, p. 362, particularly Theorem 1 and Corollary 1, for the explicit derivation of the ri-identity):\(^1\)

\[
(\sum_{t=1}^{T} p_t (d_t I + r b_{t-1} I) = I)
\]
(i) The profit calculation must obey the accounting identity \( b_t = b_{t-1}I - d_tI \): all prior year adjustments, extraordinary items and asset revaluation surpluses are passed through the profit and loss account. Hence, all current cost accounting holding gains and losses affecting the accounting value must be included in the profit.

(ii) Any opening error in asset valuation equals the closing error, which is the case for completed projects where errors are zero \( b_0I = I \) and \( b_TI = 0 \).

If both these conditions are met, the investment is allocated such that the present value of the allocations equals the initial investment. Such allocation rules are called complete (see, for example, Rogerson, 1997, p. 779; as well as Bromwich and Walker, 1998, p. 406).

One important issue of the ri-identity is that it holds true for all possible depreciation schedules. In other words, the construction of the residual income measure is independent of the depreciation schedule selected, or as Peasnell puts it: 'Any depreciation will do' (Peasnell, 1981, p. 365, italics in original). Accounting has always been interested in such accruals. The reason is that otherwise the allocation rule must be restated for all possible depreciation schedules, which seems rather expensive (see, for example, Peasnell, 1981, p. 361, for the advantages of multiple depreciation possibilities).

Despite the residual income concept, economic research has long recognised the dangers of inputting investment decisions to accounting profit rates, because this might lead to intertemporal distortions of the investment decision due to the depreciation schedules selected (see, for example, Solomons, 1965; and Peasnell, 1982).

Recently, residual income concepts marketed by Stern Stewart & Co. under the label Economic Value Added (EVA) and by McKinsey & Company, Inc. under the name Economic Profit (EP) have undergone a strong resurgence. One main argument for the use of these concepts is their compatibility with the net present value rule. A variety of similar concepts based on cash flows or residual income measures have also been marketed by other consulting companies or have already been implemented by companies (see, for example, Stewart, 1991; Bromwich and Walker, 1998; and O’Hanlon and Peasnell, 1998). Our research questions associated with the observation of such value based management concepts are the following:

(i) Are there other, undiscovered and undiscussed, investment criteria based on available accrual accounting numbers that enable net present value consistent investment decisions, independent of the depreciation schedule selected?

(ii) Which accounting conditions must the accrual accounting numbers satisfy? In particular, what opening and closing conditions must be fulfilled? Also, must the allocation rule, like the residual income identity, be complete?

The purpose of this article is to characterise the entire class of all such investment criteria.

This paper is organised as follows: Section 2(i) presents the basic model structure and basic definitions. Section 2(ii) characterises the class of all net present value-consistent investment criteria that are generated by currently available accrual accounting numbers. Section 3 completes the paper with a short discussion of the findings.

(ii) Related Literature

As mentioned earlier, Preinreich (1938) was the first to demonstrate that the present value of residual income is equivalent to the net present value of a project. Interestingly, Preinreich's finding has been reanalysed more carefully by several authors over several decades. As an important consequence, it...
became clear that one major impact of the ri-identity is that it is true for any project, any book value and any method of depreciation, as long as the book values are calculated according to the accounting identity, the hurdle rates are set equal to the capital cost rates and the initial investment is written off entirely (see Edwards and Bell, 1961; Solomons, 1965; Kay, 1976; and Peasnell, 1982).

One main condition, under which the ri-identity is fulfilled, is that the net present value and the present value of the residual incomes are calculated over the entire life of a project. Scapens (1978 and 1979) demonstrated in a dynamic, neo-classical model that under certain conditions the myopic short-run maximisation of periodic economic profit will lead to the same decision as the maximisation of long-run wealth (for similar conclusions see Tomkins, 1975; and Emmanuel and Otley, 1976). Recent research based on this idea from Anctil (1996) as well as Anctil, Jordan and Mukherji (1997) has shown with a dynamic model, in which information is decentralised and communication costs increase with the complexity of messages, that if managers maximise periodic residual income, this converges at the limit to maximum long-run wealth. Overall, these analyses have shown that periodic residual income can serve as an approximation of the net present value if problems of periodic inconsistency arise.

In the same vein, in the so-called asset base debate, academics have been aware that residual income's consistency with the npv-rule holds only when the comparison is made over the entire life of a project. Single periodic residual incomes are typically not consistent with the npv-rule. For example, the straight-line depreciation method can give positive residual income ex-ante in the first years of a project's life despite the project having a negative net present value (see, for example, Egginton, 1995, p. 204, p. 217). To solve such problems, appropriate depreciation schedules were developed to design periodically consistent residual incomes. Two suggested measures are of primary importance: earned economic income, proposed by Grinyer (1985) and maintainable residual income, suggested by Egginton (1995) (for an overview see Peasnell, 1995; as well as Bromwich and Walker, 1998, p. 402). The earned economic incomes are constructed such that they are proportional to the cash flows and the net present value of the project at each period of time. Maintainable residual income selects depreciation so that the residual income in each period is the same (see Egginton, 1995; Bromwich and Walker, 1998, p. 401; and O’Hanlon and Peasnell, 1998, p. 433). Rogerson (1997) and Reichelstein (1997 and 2000) were the first to analyse, within a scenario of incomplete information, the problem of an impatient manager characterised by the following points:

(i) Within a company, a divisional manager makes a decentralised investment decision and headquarters provide the capital.

(ii) Unlike the headquarters, the divisional manager knows the cash-flow structure for the investment project at the time of decision (ex-ante). Headquarters can observe cash flows only ex-post, but has some knowledge of cash flow patterns.

(iii) The impatient manager discounts future payments at a higher calculated interest rate than headquarters. Headquarters does not know the discount factors of the divisional management.

Within this setting, the question analysed is how to set up an incentive scheme to achieve goal-congruent investment decisions between the manager and headquarters when the class of contracts is restricted to disaggregated contracts that do not provide certain forward looking project information. Rogerson (1997) and Reichelstein (1997 and 2000) have shown that residual incomes can, under certain conditions, act as appropriate substitutes. Thereby, the necessary information is integrated into the performance measure by what is known as the relative benefit depreciation schedule, which assures the allocation rule is proportional to the relative growth profile of the investment project (see the findings of Rogerson, 1997, Proposition 1 and 4; and Reichelstein, 1997, Proposition 2 and 3; as well as the overview by Bromwich and Walker, 1998, p. 409). The relative benefit depreciation schedule is, interestingly, equivalent to the depreciation schedule selected in
earned economic incomes and for the special case of constant payments, it is equivalent to the maintainable residual income (see Bromwich and Walker, 1998, p. 407). Additionally, the project information can be incorporated into the residual income measure via capital costs rates (see Pfeiffer, 2000).

As mentioned earlier, the main rationale for the use of residual income is its consistency with the npv-rule. The literature on real options has pointed out that the rule 'invest if-and-only-if the net present value of the project exceeds zero' changes 'to invest if-and-only-if the net present value of the project exceeds the value of the option to wait' (see, for example, Dixit and Pindyck, 1994; and Stark, 2000, p. 313). Kay and Mayer (1986), Grinyer and Walker (1990) and Stark (2001) analyse performance measures that support investment decision-making in a real options context. In particular, Stark (2001) shows that a residual income-type measure supports optimal investment and disinvestment decision-making. One problem with this solution, which is true for almost all solutions, is that a circularity problem arises by calculating the cost of capital, which means that information of the entire project is needed ex-ante to calculate the costs of capital. Hence, it is possible to make the investment decision without calculating the residual income measure (see Stark, 2001, p. 325).

Theoretically the circularity problem of calculating the costs of capital also arises if several interdependent investment projects with binding budget constraints are considered. The optimal costs of capital are calculated according to the rule 'capital cost rates plus opportunity costs' where the opportunity costs are the Lagrangian multipliers associated with the budget constraints. To derive the Lagrangian multipliers, the investment problem has to be solved. Thus, a circularity problem occurs. Such a circularity problem also arises for single projects when interior solutions do not exist, because in these cases the upper-and lower-boundaries also have to be considered using Lagrangian multipliers for the boundaries (see, for example, Tomkins, 1973, Chapters 6 and 7; and Amer, 1969b, Chapter 4). In both cases, the technical reason for the circular result is the primal-dual circularity problem of constraint optimisation problems. Even if we consider single projects with an interior solution, then the circularity problem can arise because, in theory, the investment decision must already have been known in order to calculate the appropriate cost of capital (see also, the example of Stark, 1986, p. 22). These problems, for example, can be overcome under certain capital market conditions, such as a perfect market resulting in a separation of investment decisions and the determination of capital cost rates (see, for example, Magill and Quinzii, 1996, Chapters 3 and 6). However, this is not a practical problem to the extent that firms typically use exogenously determined approximations of the costs of capital (see, for example, Stewart, 1991, Chapter 12; Copeland, Koller and Murrin, 2000, Chapter 10; and Stark, 2000, p. 327).

Additionally, investment criteria based on the return on investment concepts have been analysed for investment decision-making. Of particular interest for research has been the accounting rate of return; the ratio of the accounting profit of the period and the book value of assets at the beginning of the period. Considerable effort has been invested in research to analyse conditions under which the accounting rate of return reconciles with or deviates from the economist's concept of the internal rate of return (see Stauffer, 1971; Salamon, 1973; Kay, 1976; Peasnell, 1982; and Gordon and Stark, 1989). Stauffer (1971) and Gordon and Stark (1989), for example, analysed the sign and magnitude of the difference between the accounting rate of return and the internal rate of return and under which depreciation schedule both are equal. In the same vein, Kay (1976) developed a weighted averaging scheme of return on investments that equals the internal rate of return. Drawbacks of this scheme were that the weights depended on the internal rate of return and that accounting valuation errors could appear (see Kay, 1976 and 1978; Wright, 1978; and Peasnell, 1982, p. 370). Furthermore, Kay (1976), using a continuous-time framework, and Peasnell (1982), analysing a discrete-time model, showed that the present value of cash flows discounted at the accounting rates of return are exactly equal to the initial investment if there are no opening and closing valuation errors.

Our analysis differs from the literature mentioned above on several points. Like the literature on the ri-identity, we consider an investment decision-making problem over the entire life of a single project. We do not analyse time-inconsistency problems such as the literature on the asset base debate or the
problem of the impatient manager. Furthermore, we do not model problems of real options. Finally, we consider capital cost rates as exogenously given. Our setting is consistent with the literature on the ri-identity. This is due to our research question stated above. In particular, our analysis differs from the ri-analyses and RoI-analyses in the following points:

(i) **Structure of npv-consistent performance measures:** First, we start with a broad class of accrual accounting numbers and analyse which of these performance measures enable npv-consistent investment decisions. We explicitly do not assume, like the literature on the ri-identity, that the accrual accounting numbers must be residual income. As a result, we show *endogenously* that the performance measures must have a residual income structure (see Condition (R) of Proposition 1).

(ii) **Hurdle rates:** Given the result concerning the structure of performance measures, we show *endogenously* how the hurdle rates must be determined for residual incomes (see Condition (H) of Proposition 1). The ri-analyses have shown that if the hurdle rates are set equal to the capital cost rates, then the ri-identity holds true. However, these studies have not analysed whether there are other possibilities calculating the hurdle rates, for example for the case when another depreciation basis is used.

(iii) **Closing and opening errors:** According to the ri-analyses, the initial investment is documented in the opening book value and then written off entirely over the project’s life to avoid opening and closing errors. Our analysis shows *endogenously*, how to generalise these opening and closing error conditions (see Condition (C) of Proposition 1).

In summary, we generalise the analyses concerning the ri-identity by endogenously determining the structure of all npv-consistent investment criteria, the hurdle rates, the closing and opening error conditions.

2. The Model

(i) **Assumptions and Definitions**

In the following, we consider an investment project $P$ for $T$ periods. The project generates cash in- and outflows of:

$$P = (-I, c_1(I), \ldots, c_T(I))$$

where $I \geq 0 (I \in \mathcal{I})$ is the level of investment and $c_t(I)$ the cash-flow function at time $t$. The investment project is one of all projects $P \in \mathcal{P}$ available to the decision maker and is selected at random from nature ($P \in \mathcal{P}$). Additionally, an accounting system associated with the investment project is considered. The system tracks the realised ex-post cash flows $[I$ and $c_t(I)]$ at time $t$. In addition to pure cash flows, the system also determines accrual accounting numbers as *depreciation* $[d_t(I)]$ and *book values* $[b_t(I)]$ at time $t (t = 1, \ldots, T)$:

$$b_0I = \gamma I, \quad b_{t-1}I = b_tI - d_tI = \left(\gamma - \sum_{i=1}^{t} d_i\right)I \quad (d = (d_1, \ldots, d_T) \in \mathcal{D} \subseteq \mathbb{R}_+^T)$$

To explicitly analyse opening and closing conditions, we assume that the initial book value $b_0I$ equals $\gamma$-times multiplied by the initial investment $I$ whereby $\gamma$ is the degree to which the initial investment is recorded ($\gamma > 0$). Consequently, values other than the initial investment can be used as a depreciation basis, such as the reacquisition values often used in management cost accounting. If the chosen investment level of the project is depreciated completely over $T$ periods ($b_TI = 0$, respectively $\sum_{t=1}^{T} d_t = \gamma$), the closing errors will be 0. In the following, we only assume that there is real depreciation $d_t > 0$ for at least one point in time ($\mathcal{D} \subseteq \mathbb{R}_+^T$) and that the book values are calculated according to the accounting identity ($b_tI = b_{t-1}I - d_tI$).
In the following, we want to analyse the class of all accrual accounting numbers $\Pi(\cdot)$ based on currently available accounting data at time $t$: realised accounting profits $[0, \text{respectively, } c_t - d_t I]$ and book values $[b_t, I]$:

$$\Pi(I) - dI, b'|d| = (\Pi_1(c_1(I) - d_1 I, b_0 I|d|), \ldots, \Pi_T(c_T(I) - d_T I, b_{T-1} I|d|)$$

($c(I) = (c_1(I), \ldots, c_T(I))$ and $b = (b_0, \ldots, b_{T-1})$). We assume, that all functions $\Pi(\cdot)$ are continuously differentiable in all arguments ($t = 0, \ldots, T$). Our class of accruals includes typically used performance measures such as accounting profits, residual incomes and return on investment measures, for example, residual incomes per investment. Table 1 shows how these measures can be constructed.

Typically, investment projects are evaluated according to the npv-rule. Hence, the decision maker sets the level of investment to maximise the net present value $\text{NPV}(I)$ of the project as follows:

$$\text{max}\left\{\text{NPV}(I) = \sum_{i=1}^{T} b_i c_i(I) - I | I \in \mathcal{I}\right\}$$

The net present value rule is—as the name says—an investment criteria generated by pure cash flows $[\Pi_0(\cdot) = -I$ and $\Pi_t(\cdot) = c_t(\cdot), t = 1, \ldots, T]$. In the following, we want to expand this criteria by defining investment criteria based on a system of accruals. For example, the present value of residual income measures is, according to our definition, an investment criteria generated by residual income measures. Furthermore, we want to characterise the class of all investment criteria based on accrual accounting numbers that lead to the same investment decisions as the npv-rule for all investment projects and all depreciation schedules considered.

**Definition 1:** In the following, we make the subsequent two definitions:

(i) We say an investment criterion $\Psi(\cdot)$ is generated by the system of accruals $\Pi(\cdot) = (\Pi_0(\cdot), \ldots, \Pi_T(\cdot))$ if-and-only-if the investment project $P \in \mathcal{P}$ is evaluated according to the present value of the accruals:

$$\Psi(I|d) = \sum_{i=1}^{T} \Pi_i(c_i(I) - d_i I, b_{i-1} I|d|$$

(ii) An investment criterion $\Psi(\cdot)$ generated by the system of accruals $\Pi(\cdot)$ enables npv-consistent investment decisions for all investment projects $P$ independent of the depreciation schedule $d$ selected if-and-only-if:

arg $\max\left\{\text{NPV}(I)|I \in \mathcal{I}\right\} =$

arg $\max\left\{\Psi(I|d)|I \in \mathcal{I}\right\}$ $\forall d \in \mathcal{D}, \forall P \in \mathcal{P}(A)$

is satisfied.

Condition (A) says that the investment criterion must lead to the same investment decision as the npv-rule for all investment projects considered. Condition (A) does not require that the criterion is equivalent to the net present value of the project. For example, every strictly monotone increasing function of the npv-rule also enables npv-consistent investment decisions. Additionally, Condition (A) ensures that the criterion leads to the same investment decisions for all depreciation schedules selected.

**(ii) Constructing NPV-Consistent Investment Criteria**

In this section, we analyse general investment projects $(-b, c_1(I), \ldots, c_T(I))$ where there are no assumptions of regularity for cash-flow functions—for example, strictly monotone-increasing and concave functions. The following proposition provides an if-and-only-if characterisation of the class of all investment criteria generated by the system of available accruals that enable npv-consistent investment decisions. (The proof of Proposition 1 is given in the Mathematical Appendix).
Proposition 1: The class of all investment criteria generated by available accounting measures satisfying (A) is up to a linear transformation \( \alpha \Psi^{RI}(\cdot) + \beta \) given by residual incomes \((\alpha \in \mathbb{R}_{++} \text{ and } \beta \in \mathbb{R})\)

\[
\Psi^{RI}(I|d) = \sum_{t=1}^{T} t(c_t(I) - d_tI - r_{ht-1}I)
\]

whereby the hurdle rates must be set equal to:

\[
r_{h1} = \frac{1 + r - \gamma}{\gamma} \quad \text{and} \quad r_{ht} = r \\
(t = 2, \ldots, T)
\]

and the following closing and opening error condition:

\[
b_0I = \gamma I \quad \text{and} \quad b_TI = 0 \quad \text{(or equivalent} \quad \sum_{t=1}^{T} d_tI = \gamma I) 
\]

must be satisfied.

Proposition 1 shows how investment criteria based on accrual accounting numbers can be constructed such that they enable npv-consistent investment decisions for all depreciation schedules. First of all, we want to discuss the (technical) structure of Proposition 1. Proposition 1 shows that the following three conditions must be fulfilled to obtain such npv-consistent investment criteria:

(i) **(R)**esidual income structure of the accrual accounting numbers: The structure of the performance measures must be residual incomes. Although we have considered a much richer class of accrual accounting numbers, for example, including return on investment numbers, Proposition 1 shows that there are no performance measures other than residual incomes that enable npv-consistent investment decisions for all depreciation schedules.

(ii) **(H)**urdle rates: The hurdle rates must be calculated according to (H). If, like in the ri-identity literature, opening booking value equals exactly the initial investment, then according to Condition (H) the hurdle rates must equal the capital cost rates. Thus, Proposition 1 shows that this condition is also necessary, not only sufficient. Furthermore, Condition (H) is a generalisation of how to calculate the hurdle rates when another depreciation basis is used.

(iii) **(C)**losing and opening error condition: Condition (C) states that there should be no closing error, requiring that the projects must be written off entirely. But Condition (C) does not require that a depreciation basis other than the initial investment must be used.

Because Proposition 1 provides an if-and-only-if characterisation of the class of investment criteria generated by currently available accounting measures, it is not possible to construct other performance measures not characterised in Proposition 1 that fulfil Condition (A). Or to put it the other way round, if one of these three conditions is not satisfied, then it is impossible to construct such npv-consistent investment criteria. Although Condition (A) does not require that the criterion must be equivalent to the net present value of the project, it is important to note that the net present value and the present value of residual incomes are identical (see therefore Part I of the proof). The identity:

\[
\text{NPV}(I) = \Psi^{RI}(I|d) \quad \forall d \in \mathcal{D}, \forall P \in \mathcal{P}
\]

holds true for any project, any book value and any depreciation, as long as the book values are calculated according to the accounting identity, the hurdle rates are constructed according to Condition (H) and the Closing and Opening Error Condition (C) is satisfied. Thus, we need not explicitly consider regularity conditions for the investment problem, because if there is no solution to
the investment problem based on the npv-rule, then there is no solution based on residual incomes (and vice versa).

Secondly, we want to discuss the impact of Proposition 1 on accounting profits and residual incomes. Proposition 1 shows that the only possibility to correct accounting profits such that they are npv-consistent for all depreciation schedules, is to use residual income measures. Although the analysed class of investment criteria generated by accrual accounting numbers have been enriched by more common non-linear valuation functions, there is no non-linear generalisation of accounting profit that enables (A). This result highlights the problems and limitations of profit as an investment criterion. Furthermore, if residual income measures are then considered up to a linear transformation only, the present value of residual income measures enables npv-consistent investment decisions, if-and-only-if the initial investment is written off entirely and the hurdle rates are set equal to the capital cost rates. Hence, Proposition 1 expands the analysis of Peasnell (1981 and 1982) by showing that both these conditions are not only sufficient but also necessary conditions and by showing how to adopt the hurdle rates if other depreciation bases are used (see, in particular, Peasnell, 1981, p. 53 and 1982, p. 362, Theorem 1). Furthermore, the result shows that if these two conditions are not satisfied, it is impossible to construct npv-consistent residual income measures, for example, other hurdle rates and another closing error condition.

Thirdly, we want to discuss the impact of Proposition 1 on the allocation rules used and the closing error condition. Typically in accounting, complete allocation rules are used. In the same vein, Proposition 1 shows that the investment must be allocated over time such that the present value of the allocations is exactly the same as the initial investment and not the opening book value, if a depreciation basis other than the initial investment is used:

\[
\sum_{t=1}^{T} p_t (d_t I + r_{t|t} b_{t-1} I) = I \neq \gamma I = b_0 \quad (\text{for } \gamma \neq 1)
\]

Proposition 1 shows that it is not possible to construct, via non-linear performance measure functions, any other npv-consistent investment criteria based on a non-complete allocation rule. Thus, our result provides a kind of rationale for the concept of complete allocation rules.

Finally, Proposition 1 can be interpreted as an impossibility result, because it is an if-and-only-if characterisation of the entire class of all npv-consistent investment criteria based on currently available accrual accounting numbers. As mentioned before, Proposition 1 shows the only possibility to generalise the hurdle rates and closing and opening error conditions (see Condition (C) and (H) of Proposition 1). Furthermore, Proposition 1 rules out other performance measures that do not have the structure of residual income (see Condition (R) of Proposition 1).

3. Conclusions

Much recent discussion has focused on the question of which investment criteria, such as the economic value added method based on residual income, enable net present value-consistent investment decisions. Studies by Preinreich (1938), Hicks (1946), Edwards and Bell (1961), Kay (1976) and Peasnell (1981 and 1982) have shown the identity of the net present value rule and the present value of residual incomes for any project, any book value and any method of depreciation. This result has become the basis for modern value based management. Our research started with the question of how this identity can be generalised. For the first time, we derived an if-and-only-if characterisation of all investment criteria generated by non-linear functionals of accounting profits and book values which enable net present value-consistent investment decisions that are independent of the depreciation schedule selected.

Proposition 1 has shown that the identity between net present value and the present value of the residual incomes, developed from accounting literature, cannot be further expanded up to a linear transformation by more common valuation functions. Hence, our analysis provides a rationale for the widespread practice of using residual income measures as a net present value-consistent investment
criterion. Furthermore, although the analysed class of investment criteria generated by residual income measures has been enriched by strictly increasing valuation functions and generalised allocation rules, there is no non-linear generalisation of accounting profit that enables net present value-consistent investment decisions independent of the depreciation schedule selected. The only possibility to avoid such problems is to expand accounting profits to residual incomes. Most of today’s consulting firms promote the economic value added method, which is already used by many companies. The paper demonstrates that only these criteria enable net present value-consistent investment decisions independent of the depreciation schedule selected. This also might explain why other methods based on accrual accounting numbers are not used in practice.

In summary, our analysis provides new insights into the concept of residual income as a net present value-consistent investment criterion and the interaction of exceedingly important concepts of accounting: residual income measures, closing and opening error conditions, hurdle rates and allocation rules. Proposition 1 can be interpreted as an impossibility result, making it possible to provide insights about the limits of how these concepts can be generalised.

**Mathematical Appendix**

**Proof:** Part I. First, we show that if the class of all investment criteria satisfy the conditions of Proposition 1, then (A) is fulfilled. Using \( d_t = b_{t+1} - b_t \), we get:

\[
\sum_{t=1}^{T} p_t (d_t I + r_{ht-1} b_{t-1} I) = \sum_{t=1}^{T} b_t ((b_{t-1} - b_t) I + r_{ht-1} b_{t-1} I)
\]

\[
= \sum_{t=1}^{T} p_t ((1 + r_{ht-1}) b_{t-1} I - b_t I) = \sum_{t=0}^{T-1} p_{t+1} (1 + r_{ht}) b_t I - \sum_{t=1}^{T} p_t b_t I
\]

Applying \( r_{h0} = ((1 + r) / \gamma) \), \( r_{ht} = r \), \( b_0 = \gamma I \) and \( b_T = 0 \) leads to:

\[
p_1 (1 + r_{h0}) b_0 I + \sum_{t=1}^{T-1} p_{t+1} (1 + r_{ht}) b_t I - \sum_{t=1}^{T-1} p_t b_t I - p_T b_T I
\]

\[
= p_1 \left(1 + \frac{1 + r - \gamma}{\gamma}\right) \gamma I + \sum_{t=1}^{T-1} p_{t+1} (1 + r) b_t I - \sum_{t=1}^{T-1} p_t b_t I - p_T b_T I
\]

\[
= I - p_T b_T I = I
\]

Finally, we get for positive linear transformations \( (\alpha > 0) \):

\[
\max \{\alpha \Psi^{R I}(I) + \beta = \alpha \sum_{t=1}^{T} p_t (c_t(I) - d_t I - r_{ht-1} b_{t-1} I) + \beta I \in I\}
\]

\[
= \alpha \max \left\{\sum_{t=1}^{T} p_t c_t(I) - I = \text{NPV}(I) | I \in I\right\} + \beta
\]

Obviously, these calculations are true for all investment projects and all depreciation schedules considered. This completes Part I of the proof.

Part II. We assume that another npv-consistent investment criterion \( \Psi(\cdot) \) exists for all possible investment projects. In particular, this investment criterion \( \Psi(\cdot) \) must be npv-consistent for all differentiable investment projects, for which the npv-investment level is given by its first-order condition (Foc1):

\[
0 = \frac{\partial \text{NPV}(I)}{\partial I} = \sum_{t=1}^{T} p_t \frac{\partial c_t(I)}{\partial I} - 1
\]
Due to the differentiability of the performance measures $\Pi_t(\cdot)$, the investment criterion $\Psi(\cdot)$ is also differentiable and the first-order condition of the investment criteria must also hold at the npv-investment level (Foc2):

$$0 = \frac{\partial \Psi(\cdot)}{\partial I} = \sum_{t=1}^{T} p_t \left( \frac{\partial \Pi_t(\cdot)}{\partial (c_t - d_tI)} \left( \frac{\partial c_t(I)}{\partial I} - \frac{\partial d_tI}{\partial I} \right) + \frac{\partial \Pi_t(\cdot)}{\partial b_{t-1}I} \frac{\partial b_{t-1}I}{\partial I} \right)$$

Multiplying (Foc1) with $1/\alpha > 0$ and applying (Foc2) leads to:

$$\frac{1}{\alpha} \left( \sum_{t=1}^{T} p_t \left( \frac{\partial \Pi_t(\cdot)}{\partial (c_t - d_tI)} \left( \frac{\partial c_t(I)}{\partial I} - \frac{\partial d_tI}{\partial I} \right) + \frac{\partial \Pi_t(\cdot)}{\partial b_{t-1}I} \frac{\partial b_{t-1}I}{\partial I} \right) \right) = \sum_{t=1}^{T} p_t \frac{\partial c_t(I)}{\partial I} + 1$$

Rearranging leads to:

$$0 = \sum_{t=1}^{T} p_t \frac{\partial c_t(I)}{\partial I} \left( \frac{1}{\alpha} \frac{\partial \Pi_t(\cdot)}{\partial (c_t - d_tI)} - 1 \right) - \left( \frac{1}{\alpha} \left( \sum_{t=1}^{T} p_t \left( \frac{\partial \Pi_t(\cdot)}{\partial (c_t - d_tI)} \frac{\partial d_tI}{\partial I} - \frac{\partial \Pi_t(\cdot)}{\partial b_{t-1}I} \frac{\partial b_{t-1}I}{\partial I} \right) \right) \right)$$

This equation must be satisfied for all possible investment levels $I \in I$, if we consider a class of parametric investment projects $(P(y) \in P, y \in Y)$ for which the npv investment level $I(y) \in I$ varies with the parameter $y \in Y$ over the entire interval $I$. Hence, the equation is a polynomial of degree one in the variables $(y_0, \ldots, y_T) = (-I, \partial c_t(I)/\partial I, \ldots, \partial c_T(I)/\partial I)$, which must be identically zero for all $(y_0, \ldots, y_T)$ (because, by assumption, the investment criterion rule cannot be conditioned on the class of investments considered). According to the Fundamental Theorem of Algebra, this is true if-and-only-if all coefficients are set equal to zero. This leads to (Co1) and (Co2):

$$\frac{\partial \Pi_t(\cdot)}{\partial (c_t - d_tI)} = \alpha (\forall y \in Y, \forall d \in D)$$

$$\sum_{t=1}^{T} p_t \left( \frac{\partial \Pi_t(\cdot)}{\partial (c_t - d_tI)} d_t - \frac{\partial \Pi_t(\cdot)}{\partial b_{t-1}I} b_{t-1} \right) = \alpha (\forall y \in Y, \forall d \in D)$$

which must be satisfied for all investment projects and all depreciation schedules.

To analyse (Co2), we rewrite it as follows $(\partial c_t(I)/\partial (c_t - d_tI) = \alpha$ and $I = b_0/\gamma)$:
\[ 0 = \frac{1}{\alpha} \left( \sum_{t=1}^{T} p_t \left( \frac{\partial \Pi_t(\cdot)}{\partial I} \cdot d_t - \frac{\partial \Pi_{t-1}(\cdot)}{\partial b_{t-1}I} \cdot b_{t-1} \right) \right) + I \]

\[ = \frac{1}{\alpha} \left( \sum_{t=1}^{T} p_t \left( \alpha d_t - \frac{\partial \Pi_t(\cdot)}{\partial b_{t-1}I} \cdot b_{t-1} \right) \right) + \frac{1}{\gamma} b_0 \]

\[ = \sum_{t=1}^{T} p_t \left( b_{t-1} - b_t \right) - \frac{1}{\alpha} \frac{\partial \Pi_{t-1}(\cdot)}{\partial b_{t-1}I} \cdot b_{t-1} + \frac{1}{\gamma} b_0 \]

\[ = \sum_{t=1}^{T} p_t \left( 1 - \frac{1}{\alpha} \frac{\partial \Pi_{t-1}(\cdot)}{\partial b_{t-1}I} \right) b_{t-1} - \sum_{t=1}^{T} p_t b_t + \frac{1}{\gamma} b_0 \]

\[ = \left( p_1 \left( 1 - \frac{1}{\alpha} \frac{\partial \Pi_{t-1}(\cdot)}{\partial b_1I} \right) - \frac{1}{\gamma} \right) b_0 + \sum_{t=1}^{T-1} \left( p_{t+1} \left( 1 - \frac{1}{\alpha} \frac{\partial \Pi_{t+1}(\cdot)}{\partial b_{t+1}I} \right) \right) b_t - p_T b_T \]

This equation can be restated with the following matrix \( A \in R^{(T+1) \times (T+1)} \)

\[
\begin{bmatrix}
-p_T & 0 & \cdots & \cdots & 0 \\
0 & p_T & \cdots & \cdots & 0 \\
\vdots & \vdots & \ddots & \cdots & \vdots \\
0 & \cdots & \cdots & p_T & 0 \\
0 & \cdots & \cdots & \cdots & 0 \\
\end{bmatrix}
\]

and the vector \( b = (b_0, \ldots, b_T) \in R^{T+1} \) as follows: \( Ab = 0 \). This equation must be satisfied for all possible depreciation schedules \( d \)-respectively for all possible book values \( b \): \( Ab = 0 \) \( \forall b \). According to the Fundamental Theorem of Algebra, this is true if-and-only-if \( A = 0 \in R^{(T+1) \times (T+1)} \). Hence, we have \( A = 0 \) if-and-only-if:

\[ b_T = 0, \quad \frac{\partial \Pi_1(\cdot)}{\partial b_0I} = -\frac{1 + r - \gamma}{\gamma} \quad \text{and} \quad \frac{\partial \Pi_t(\cdot)}{\partial b_{t-1}I} = -\alpha r \]

are satisfied \( (t = 2, \ldots, T) \).

In summary, the following equations must be satisfied for all possible book values \( b \) and the considered class of investment projects:

\[ \frac{\partial \Pi_1(c_1(I) - d_1I, b_0I)}{\partial c_1(I) - d_1I} = \alpha \quad \text{and} \quad \frac{\partial \Pi_1(c_1(I) - d_1I, b_0I)}{\partial b_0I} = -\frac{1 + r - \gamma}{\gamma} \]

\[ \frac{\partial \Pi_t(c_t(I) - d_tI, b_{t-1}I)}{\partial c_t(I) - d_tI} = \alpha \quad \frac{\partial \Pi_t(c_t(I) - d_tI, b_{t-1}I)}{\partial c_t(I) - d_tI} = -\alpha r \]

\( (t = 2, \ldots, T) \). Hence, the first derivatives must be constant in all arguments over the considered interval. The only class of all differentiable functions where all first derivatives are constant over an entire interval, is the class of linear functions. Hence, the functions must be linear with the structure:

\[ \Pi_1(c_1(I) - d_1I, b_1I) = \alpha \left( c_t(I) - d_tI - \frac{1 + r - \gamma}{\gamma} b_0 \right) \]

\[ \Pi_t(c_t(I) - d_tI, b_{t-1}I) = \alpha(c_t(I) - d_tI - \gamma b_{t-1}) \]

\( (t = 2, \ldots, T) \). This completes the proof. □
Footnotes

1 Peasnell (1982, p. 362, Accounting Identity 1) states that accounting profits must equal the net dividends plus the change in net book values during the period. In this definition, accounting profits are of the all inclusive or clean surplus variety (for the clean surplus concept see Feltham and Ohlson, 1995). We do not explicitly want to restrict our analysis to the valuation of equity, thus we do not consider the question of dividends any further. Hence, our accounting identity, which is technically the same, has a slightly different interpretation (on this point see Peasnell, 1982, Point (i)). It is important to note that both definitions do not explicitly consider prepayments, inventories, etc. (for a more general definition of the depreciation concept, see, for example, Stauffer, 1971; and Gordon and Stark, 1989).

References


Solomons, D. (1965), Divisional Performance: Measurement and Control (Homewood (Ill.).)


